Benha University Faculty of Engineering- Shoubra Electrical Engineering Department First Year communications.		1^{st} semester Exam Date: 31-12-2012 ECE111: Electronic Engineering fundamentals Duration : 3 hours

$K=1.38\times10^{-23}$ J/K	$h=6.64\times10^{-34}$ J.s	$q=1.6\times10^{-19}$ C	$m_o=9.1\times10^{-31}$ Kg	[Si] $n_i=1.5\text{x}10^{10}$ cm-3
[Si] $m_e=1.18\ m_o$	[Si] $m_h=0.81\ m_o$	[Si] $E_g=1.12$ eV	[Si] $\mu_n=1400\ cm^2/V.s$	[Si] $\mu_p= 400\ cm^2/V.s$
$\varepsilon_o=8.85\text{x}10^{-14}$ F/cm	$\varepsilon_{rs}= 11.7$	Eg = 1.12 eV		

Solution

Question 1 (12 marks)

Answer this question in the form of table. Choose the correct answer (only one answer is accepted).

1- For intrinsic semiconductor ……….
   (a) All bonds are complete at 0 K (b) Part of valance electrons is released at high ***T***
   (c) There are some impurities added **(d) Both (a) and (b)**

2- The collision due to ………………… May change both magnitude and direction of the carrier speed
   (a) Ionized impurities **(b) Lattice vibrations**
   (c) Thermal motion (d) Drift of particles

3- As the time between collisions increased, the mobility ………………
   (a) Remains constant (b) Decreased
   (c) increased (d) is affected only by the impurities concentration

4- As the doping concentration increases above 1x1015, the mobility ……………
   (a) Remains constant (b) is affected only by the impurities concentration
   (c) increased **(d) Decreased**

5- Fick's low can describe …………….
   (a) Diffusion phenomena (b) Drift phenomena
   (c) Both drift and diffusion (d) Non of the above

6- For the fabrication of GaAs pn junction. The most common method is …………………
   (a) Diffusion (b) Evaporation
   (c) Epitaxy (d) Ion implantation

7- The linearly graded pn junction are usually made by
   (a) Diffusion (b) Evaporation
   (c) Epitaxy (d) Ion implantation

8- The pn junction depletion width varies as ………………..
   (a) $\varepsilon^{1/2}$ (b) $\varepsilon^{3/2}$
   (c) ε (d) $\varepsilon^{-1/2}$

9- As the reverse bias voltage increases, the depletion capacitance …………………
   (a) Decreases (b) Increases
   (c) becomes zero (d) Remains constant

10- For the active mode of operation of bipolar junction transistors. The ***EBJ/CBJ*** must be connected as ………
   (a) Forward/Forward (b) Reverse/Reverse
   (c) Forward/Reverse (d) Reverse/forward

11- For a bipolar junction transistor (BJT), the base region is ……………………..
   (a) Moderately doped (b) Very thin
   (c) Lightly doped **(d) Both(b) and(c)**

12- Most of the electrons in the base of an npn transistor flow
   (a) Out of the base lead **(b) Into the collector**
   (c) Into the emitter (d) Into the base supply

1	2	3	4	5	6	7	8	9	10	11	12
d	b	c	d	a	c	a	a	a	c	d	b

Question 2 **(20 marks)**

a- In a semiconductor, the Fermi level is 250 meV below the conduction band. What is the probability of finding an electron in a state **KT** below the valance band edge E_V at room temperature?

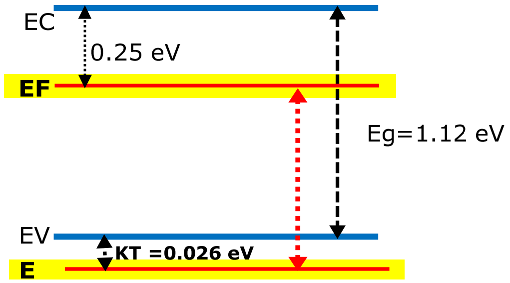
b- A bar of silicon is 0.2 mm long and has across-section of 0.2 x 0.2 mm. One volt impressed across the bar results in a current of 8 mA. Assuming that the current is due to electrons, calculate:

  i. Concentrations of free electrons and
  ii. The drift velocity.

c- The doping process of a Si changes its conductivity. There is always a certain specific doping level that causes the conductivity to be a minimum. An n-type semiconductor is doped with that specific level. Calculate the minimum value of the conductivity. (T=300 K)

Solution

(a)



$$F(E)=\frac{1}{1+e^{\frac{E-E_F}{KT}}}$$

$$E_F-E=\left(E_g-E_1\right)+KT$$

$$E_F-E=1.12-0.25+0.026=0.896\text{ eV}$$

$$E-E_F=-0.896\text{ eV}$$

$$F(E)=\frac{1}{1+e^{-\frac{0.896}{0.026}}}=\frac{1}{1+1\times10^{-15}}=1$$

(b)

$$R=\frac{V}{I}=\frac{1}{8\times10^{-3}}=125\Omega$$

$$R=\rho\frac{L}{A}$$

$$\rho=\frac{RA}{L}=\frac{125\times0.2\times0.2\times10^{-6}}{0.2\times10^{-3}}=25\times10^{-3}\Omega m=2.5\Omega cm$$

$$\sigma=\frac{1}{\rho}=0.4=q\mu_n n$$

$$n=\frac{0.4}{1.6\times10^{-19}\times1400}=1.785\times10^{15}cm^{-3}$$

$$v_d=\mu E=1400\frac{1V}{0.2\times10^{-3}\times100}=70000\ cm/\sec=700m/s$$

(c)

$$\boxed{\sigma = q\mu_e n + q\mu_h p}$$

$$\sigma = q\mu_e \frac{n_i^2}{p} + q\mu_h p$$

At minimum conductivity $\Longrightarrow$ $\dfrac{d\sigma}{dp} = 0$

$$\therefore \frac{d\sigma}{dp} = q\mu_h + q\mu_e n_i^{\,2}\left[-\frac{1}{p^2}\right]$$

$$\frac{\mathrm{d}\sigma}{\mathrm{dp}} = 0$$

$$\therefore \quad q\mu_h = \frac{q\mu_e n_i^{\,2}}{p^2}$$

$$p^2 = \frac{\mu_e}{\mu_h} n_i^2$$

$$p = n_i\sqrt{\frac{\mu_e}{\mu_h}}$$

$$p = 1.5\times10^{10}\sqrt{\frac{1400}{400}} = 2.8\times10^{10}\,cm^{-3}$$

$$np = n_i^2$$

$$n = \frac{n_i^2}{p} = \frac{\left(1.5\times10^{10}\right)^2}{2.8\times10^{10}} = 8.03\times10^{9}\,cm^{-3}$$

$$\sigma_{\min} = 1.6\times10^{-19}\left(1400\times8.03\times10^{9} + 400\times2.8\times10^{10}\right)$$

$$\sigma_{\min} = 1.6\times10^{-19}\left(1.1242\times10^{13} + 1.12\times10^{13}\right)$$

$$\sigma_{\min} = 3.6\times10^{-6}\ \Omega^{-1}\,\mathrm{cm}^{-1}$$

Question 3 **(20 marks)**

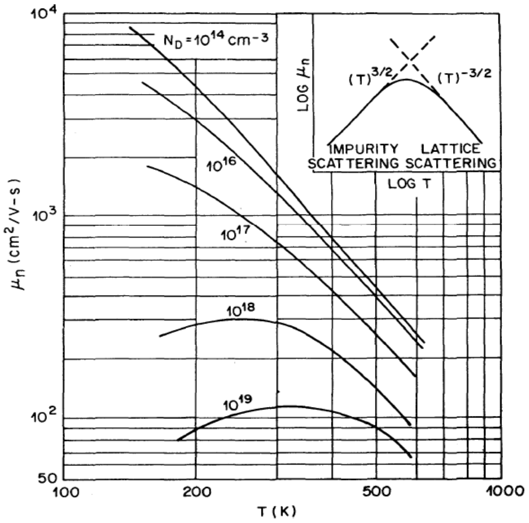
a- Explain the dependence of mobility on temperature. (not more than 6 lines)

b- Prove that: $\frac{D_e}{\mu_e} = \frac{KT}{q}$

c- A bar of silicon of length 0.4 x 10^{-3}cm is illuminated at one end creating $\Delta n = \Delta p = 10^{12}$ cm^{-3} excess electrons and holes. If the diffusion length L_p for the minority holes is 4 x 10^{-3} cm and if all the excess electrons and holes recombine at the other end of the bar. **Calculate** and **plot** the steady-state excess minority hole distribution $\Delta p(x)$ as function of the distance along the bar. (Hint Use the approximation, $e^x = 1+x$, for x << 1.)

Solution

(a)



At **high** temperatures **(T>150 K)** the mobility is mainly limited by the **lattice vibrations**. μ decreases with increase of T (**μ α $T^{-3/2}$**).

At **low** temperatures **(T<150 K)** the mobility is mainly limited by the **ionized impurities**. μ increases with increase of T (**μ α** $T^{3/2}$).

(b)

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

$$\mathbf{E} = \frac{1}{q}\frac{dE_i}{dx}$$

$$J_n = q\mu_n n\mathbf{E} + qD_n\frac{dn}{dx} = 0$$

$$q\mu_n n_i \exp\left[\frac{E_F - E_i}{kT}\right]\frac{1}{q}\frac{dE_i}{dx} = -\frac{q}{kT}D_n n_i \exp\left[\frac{E_F - E_i}{kT}\right]\left[\frac{dE_F}{dx} - \frac{dE_i}{dx}\right]$$

$$\frac{dE_F}{dx} = 0$$

Then

$$\mu_n = \frac{qD_n}{kT}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = V_T$$

(c)

Continuity equation (no light/steady state)

Note (the light is absorbed in very small region .ie at x=0 and creating an excess of minority carrier of 10^{12} cm^{-3} but the semi conductor does not expose to light

$$\boxed{\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{L_p^2} = 0}$$

This equation has a solution as:

$$\Delta p(x) = C_1 e^{\frac{-x}{L_p}} + C_2 e^{\frac{x}{L_p}}$$

Note L = 0.4×10^{-3} cm and $L_p = 4\times10^{-3}$ cm

$$\text{since } L \ll L_p \quad \therefore \frac{x}{L_p} \ll 1$$

$$\because e^x = 1 + x \qquad \text{when} \quad x \ll 1$$

Then

$$\Delta p(x) = C_1(1 - \frac{x}{L_p}) + C_2(1 + \frac{x}{L_p})$$

$$or \;\; \Delta p(x) = (C_1 + C_2) - (\frac{C_1 - C_2}{L_p})x$$

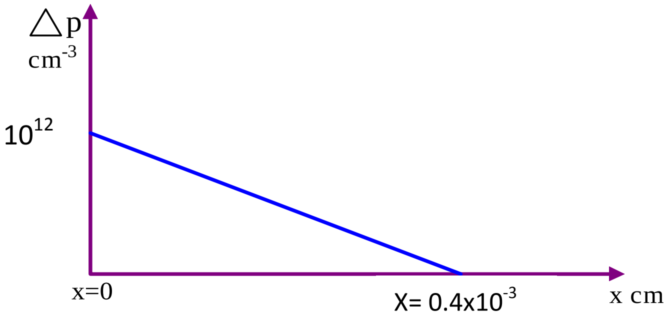
** At x = 0 $\Delta p = \Delta p_o = 10^{12}$

ie $C_1 + C_2 = 10^{12}$ I

** At x =L $\Delta p = 0$

ie $(C_1 + C_2) - 0.1(C_1 - C_2) = 0$ II

By solving (I) and (II)



$C_1 = 5.5\times10^{12}$

$C_2 = -4.5\times10^{12}$

$$\therefore \Delta p(x) = 10^{12} - 2.5\times10^{15} x$$

Thus for $x/L_p << 1$ the distribution becomes ***linear*** **not exponential** as shown

Question 4 (18 marks)

a- For the shown abrupt pn junction drive an expressions for the electric field in the region $-x_p < x < x_n$.

b- Define: the barrier potential and then derive the expression for the barrier potential (built-in potential) in terms of the doping concentration

c- An abrupt silicon pn junction at zero bias has dopant concentration of $N_A = 10^{17}$ cm^{-3} and $N_D = 5 \times 10^{15}$ cm^{-3}. T = 300 K.

  i. Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level.
  ii. Calculate the built-in potential.
  iii. Determine the peak electric field for this junction.

Solution

(a)

$$\frac{d^2\psi(x)}{dx^2} = -\frac{d\mathbf{E}(x)}{dx} = -\frac{\rho_s(x)}{\varepsilon}$$

$-x_p \le x < 0$

$$\frac{d\mathbf{E}(x)}{dx} = \frac{-qN_A}{\varepsilon}$$

$$\mathbf{E}(x) = -\frac{qN_A}{\varepsilon}x + \mathbf{E}_1$$

$$\mathbf{E}_1 = -\frac{qN_A}{\varepsilon}x_p$$

$$\mathbf{E}(x) = -\frac{qN_A}{\varepsilon}\left(x + x_p\right)$$

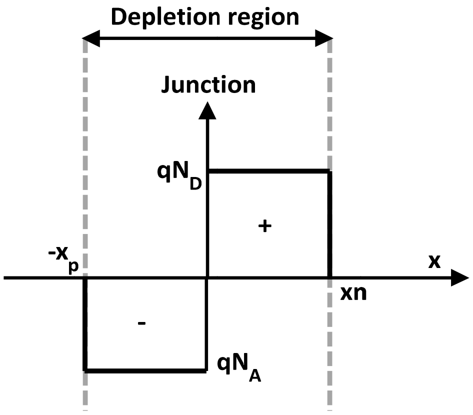
$0 < x \le x_n$

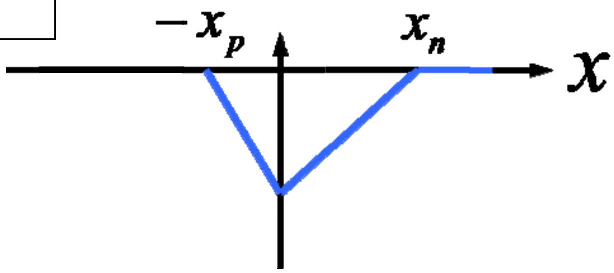
$$\frac{d\mathbf{E}(x)}{dx} = \frac{qN_D}{\varepsilon}$$

$$\mathbf{E}(x) = \frac{qN_D}{\varepsilon}x + \mathbf{E}_2$$

$$\mathbf{E}_2 = -\frac{qN_D}{\varepsilon}x_n$$

$$\mathbf{E}(x) = \frac{qN_D}{\varepsilon}\left(x - x_n\right)$$





$$\mathbf{E}_{\max} = \mathbf{E}(0) = -\frac{qN_A}{\varepsilon}x_p = -\frac{qN_D}{\varepsilon}x_n$$

(b)

Barrier potential:

when the n type material put in contact with the p type material, free electrons from n type diffuse and cross the junction and combine with holes in the p type material leaving behind (+ve ions) in the surface of the n type. While (-ve ions) on the p type reign. These positive and negative ions create an electric field which in turn produces an electric potential (barrier potential) that prevent more electrons from crossing the junction.

Derivation: straight forward until:

$$V_o = \frac{KT}{q}\ln\frac{N_D N_A}{n_i^2}$$

(c)

d- An abrupt silicon pn junction at zero bias has dopant concentration of $N_A = 10^{17}$ cm^{-3} and $N_D = 5 \times 10^{15}$ cm^{-3}. T = 300 K.

   iv. Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level.
   v. Calculate the built-in potential.
   vi. Determine the peak electric field for this junction.

n-side:

$$E_F - E_i = \frac{KT}{q}\ln\frac{n}{n_i} = \frac{KT}{q}\ln\frac{N_D}{n_i} = 0.026\ln\frac{5\times10^{15}}{1.5\times10^{10}} = 0.33eV$$

p-side:

$$E_i - E_F = \frac{KT}{q}\ln\frac{P}{n_i} = \frac{KT}{q}\ln\frac{N_A}{n_i} = 0.026\ln\frac{10^{17}}{1.5\times10^{10}} = 0.408eV$$

$$V_{bi} = \frac{KT}{q}\ln\frac{N_D N_A}{n_i^2} = 0.026\ln\frac{10^{17}\times5\times10^{15}}{\left(1.5\times10^{10}\right)^2} = 0.7391$$

$$x_n = \sqrt{\frac{2\varepsilon}{q}\frac{N_A}{N_D N_A + N_D^{\ 2}}V_{bi}}$$

$$\mathbf{E}_{\max} = \mathbf{E}(0) = -\frac{qN_A}{\varepsilon}x_p = -\frac{qN_D}{\varepsilon}x_n$$

$$x_n = \sqrt{\frac{2\times8.85\times10^{-14}\times11.7}{1.6\times10^{-19}}\frac{10^{17}}{5\times10^{32}+2.5\times10^{31}}0.73}$$

$$x_n = \sqrt{\frac{2\times8.85\times10^{-14}\times11.7}{1.6\times10^{-19}}\frac{10^{17}}{5\times10^{32}+2.5\times10^{31}}0.73} = 4.26\times10^{-5}cm = 0.426\mu m$$

$$\mathbf{E}_{\max} = \mathbf{E}(0) = -\frac{qN_D}{\varepsilon}x_n = -\frac{1.6\times10^{-19}\times5\times10^{15}\times4.26\times10^{-5}}{8.85\times10^{-14}\times11.7} = 32913.22 = 3.29\times10^4 \quad V/cm$$

Question 5 **(20 marks)**

a- Given that: $x_n = \left(\frac{2\varepsilon_s (V_{bi} + V_R)}{q} \left(\frac{N_A}{N_D} \right) \frac{1}{(N_A + N_D)} \right)^{1/2}$.

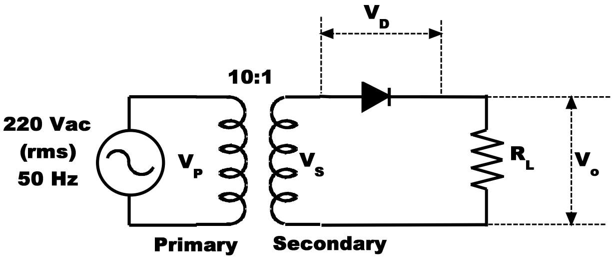
Drive an expression for the depletion capacitance of the p$^+$n, and then draw the relation between the reciprocal of the squared of the capacitance and the reverse voltage.

b- An ideal one-sided silicon p$^+$n junction has uniform doping on both sides of the abrupt junction. The doping relation is $N_A = 50\ N_D$. Given that: $V_{bi} = 0.752$V, $V_R = 10$ V, T = 300 K and the cross-sectional area of the junction is A = 5 x 10^{-5} cm^2, Determine:

  i. N_A and N_D
  ii. x_n, for $V_R = 10$
  iii. The junction capacitance.

c- A half wave rectifier with a transformer coupled input is shown in the adjacent figure

  i. Draw the waveforms V_o and V_D
  ii. Calculate the values of $V_s(p)$, $V_o(p)$, V_{averge} and F_{out} .



Solution

(a)

$$x_n = \left(\frac{2\varepsilon_s (V_{bi} + V_R)}{q} \left(\frac{N_A}{N_D} \right) \frac{1}{(N_A + N_D)} \right)^{1/2}.$$

$$for\ p^+n\ N_A >> N_D$$

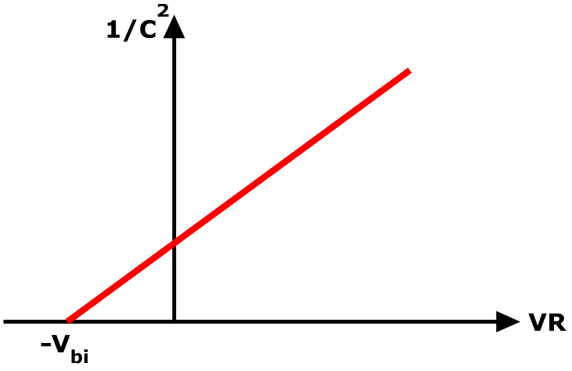
$$\therefore x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{qN_D}}$$

$$C' = \frac{dQ}{dV_R} = qN_D \frac{dx_n}{dV_R} = ...$$

$$\therefore C' = \sqrt{\frac{qN_D\varepsilon_s}{2(V_{bi} + V_R)}}$$

$$\therefore C'^2 = \frac{qN_D\varepsilon_s}{2(V_{bi} + V_R)}$$

$$\left[\frac{1}{C'}\right]^2 = \frac{2}{qN_D\varepsilon_s}(V_{bi} + V_R)$$



(b)

$$V_{bi} = \frac{KT}{q} \ln \frac{N_D N_A}{n_i^2} = 0.026 \ln \frac{50N_A^2}{(1.5\times10)^2}$$

$$0.752 = 0.026 \ln \frac{50N_D^2}{(1.5\times10)^2}$$

$$50N_D^2 = 8.19\times10^{32}$$

$$\therefore N_D = 4\times10^{15}\,cm^{-3}$$

$$N_A = 50N_D = 2\times10^{17}\,cm^{-3}$$

$$x_n = \sqrt{\frac{2\varepsilon_s\left(V_{bi}+V_R\right)}{qN_D}}$$

$$x_n = \sqrt{\frac{2\times 8.85\times 10^{-14}\times 11.7\times(0.752+10)}{1.6\times 10^{-19}\times 4\times 10^{15}}} = 1.86\times 10^{-4}\,cm = 1.86\,\mu m$$

$$\therefore C' = \sqrt{\frac{qN_D\varepsilon_s}{2(V_{bi}+V_R)}}$$

$$C' = \sqrt{\frac{1.6\times 10^{-19}\times 4\times 10^{15}\times 8.85\times 10^{-14}\times 11.7}{2(0.752+10)}}$$

$$C' = 5.551\times 10^{-9}\,F/cm^2$$

$$\therefore C = C'\times A = 5.551\times 10^{-9}\times 5\times 10^{-5} = 0.2775\times 10^{-12}\,F = 0.2775\,pF$$

(c)